A Partially Coherent CPM Receiver with Cochannel Interference *

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Abstract—**Continuous phase modulation has been investigated in recent years because of its desirable spectral properties. A partially coherent detector for CPM has previously been derived and its performance investigated for an additive white Gaussian noise channel. This paper extends the previous results by considering the effects of an interfering user. The variation of the optimum modulation index with interference power-to-signal power ratio is discussed as well.**

I. INTRODUCTION

Continuous phase modulation (CPM) is recently receiving greater research interest because of its inherent advantages over other modulation formats. CPM is a constant envelope modulation, making it ideal for use with nonlinear devices, and it has a narrow spectrum that results in a small occupied bandwidth and reduced adjacent channel interference.

Coherent demodulation of CPM uses a phase-locked-loop (PLL) to regenerate the carrier frequency and phase. It has been extensively studied in [1], [2], and [3]. In a hostile environment, such as a military application, a prime jamming strategy in a coherent system is to attack the center PLL frequency. Noncoherent systems, on the other hand, are used to deny the jammer this advantage [4]; noncoherent demodulation of CPM has been studied in [5] and [6].

The partially coherent receiver attempts to take advantage of the positive attributes of both types of receivers, coherent and noncoherent, by maintaining an estimate of quality for the carrier phase. The quality of the carrier phase may range from excellent to poor. When the quality of the phase estimate is excellent, the partially coherent receiver reduces to a coherent receiver, and when the quality of the carrier phase is poor, the partially coherent receiver reduces to a noncoherent receiver. The generation of this quality of the phase estimate is not addressed in this paper, but it can be obtained using parameter estimation theory [7] and knowledge of the state of the PLL.

Previous work with the partially coherent CPM receiver [9] did not consider the receiver performance degradation resulting from an interfering signal. The performance degradation is of interest to gauge the effectiveness of a partially coherent receiver in a multi-user environment. This paper assumes that the interfering user is using the same modulation, but has an unknown random phase, and that the interference power is less than the desired signal power, as might result from receiving the interference through antenna sidelobes.

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II. PARTIALLY COHERENT DETECTION

The full response transmitted CPM signal is a constant amplitude modulation and has the form

$$s(t,\alpha) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \phi(t,\alpha)], \qquad (1)$$

where E is the signal energy, T is the symbol period and the data is contained in the phase function

$$\phi(t,\alpha) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i q(t - iT). \qquad (2)$$

The non-negative constant h is called the modulation index and each data symbol α_i takes on one of two binary values ± 1. The pulse shape of the phase function, $q(t)$, is the integral of the full response instantaneous rectangular frequency function 1REC [3], that when normalized yields

$$q(t) = \begin{cases} 0, & t < 0 \\ \frac{t}{2T}, & 0 \le t < T \\ \frac{1}{2}, & t \ge T. \end{cases} \qquad (3)$$

The partially coherent receiver development is discussed in [8], and has been analyzed for CPM in [9]; a block diagram is shown in Fig. 1 with observation over the interval $[-N_1T, N_2T]$, where N_1 and N_2 are non-negative integers. The complexity of the receiver increases as $2^{N_1+N_2}$. However, it was shown in [9] that observation over just three symbols ($N_1 + N_2 = 3$) approaches closely the optimum detection over an infinite sequence. Consequently, this investigation will focus on using this receiver with three symbol sequence detection.

III. PROBABILITY OF ERROR

The probability of error for the partially coherent receiver is determined for detecting a desired signal while simultaneously receiving an interfering signal. The interfering signal

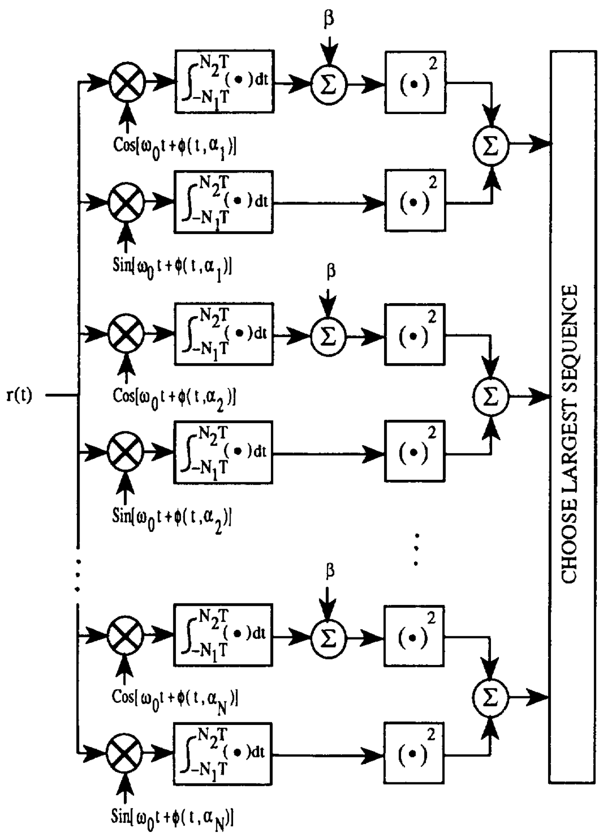

Figure 1: Binary Partially Coherent CPM Sequence Receiver, where $N = 2^{N_1+N_2}$.

has the identical modulation format (1REC) as the desired signal, but has a random data sequence ($\widehat{\alpha}$) and a random carrier phase offset (Θ) uniformly distributed over $[0, 2\pi]$. For simplicity in computation, it is assumed that the interfering signal is in time synchronization with the desired signal. Although unrealistic in practical systems, time synchronization can often be justified as a worst case scenario. The signals at the input to the receiver of Fig. 1 are then

$$r(t) = s(t,\alpha) + \sqrt{\frac{2E}{T}}\sqrt{\frac{I}{K}} cos[\omega_0 t + \phi(t,\widehat{\alpha}) + \Theta] + n(t), \quad (4)$$

where $s(t,\alpha)$ is given in Eqn. 1 and $\sqrt{I/K}$ is the interference-to-carrier power ratio that takes on real values from zero to infinity, with zero corresponding to no interference.

The receiver in Fig. 1 selects the data sequence estimate, $\widetilde{\alpha}$, of the true sequence, α, that maximizes the likelihood metric, given the interference sequence, $\widehat{\alpha}$. An error occurs when the incorrect sequence decision metric, $R(\alpha,\widetilde{\alpha})$, is larger than the correct decision metric, $R(\alpha,\alpha)$. The probability of symbol error is the probability of any possible error sequence causing an incorrect decision metric to exceed the correct decision metric. Using the union bound for this probability results in an upper bound of

$$\begin{aligned} &Pr(error \mid \widehat{\alpha},\Theta) \\ &\leq \sum_i Pr[R(\alpha,\widetilde{\alpha}_i) > R(\alpha,\alpha) \mid \widehat{\alpha},\Theta] \\ &= \sum_i Pr\left[|z_1|^2 > |z_2|^2 \mid \widehat{\alpha},\Theta\right] \\ &= \sum_i Pr[|z_1| > |z_2| \mid \widehat{\alpha},\Theta], \end{aligned} \quad (5)$$

where z_1 and z_2 are complex correlated Gaussian random variables defined from the decision metrics as

$$\begin{aligned} z_1 &\triangleq c(\alpha,\widetilde{\alpha}) + \sqrt{\tfrac{I}{K}}c_I(\widehat{\alpha},\widetilde{\alpha},\Theta) + \sqrt{\tfrac{2T}{E}}n_c(\widetilde{\alpha}) + \tfrac{1}{2}\beta \\ &+ j\left[s(\alpha,\widetilde{\alpha}) + \sqrt{\tfrac{I}{K}}s_I(\widehat{\alpha},\widetilde{\alpha},\Theta) + \sqrt{\tfrac{2T}{E}}n_s(\widetilde{\alpha})\right] \end{aligned} \quad (6)$$

and

$$\begin{aligned} z_2 &\triangleq c(\alpha,\alpha) + \sqrt{\tfrac{I}{K}}c_I(\widehat{\alpha},\alpha,\Theta) + \sqrt{\tfrac{2E}{E}}n_c(\alpha) + \tfrac{1}{2}\beta \\ &+ j\left[s(\alpha,\alpha) + \sqrt{\tfrac{I}{K}}s_I(\widehat{\alpha},\alpha,\Theta) + \sqrt{\tfrac{2T}{E}}n_s(\alpha)\right], \end{aligned} \quad (7)$$

respectively, and where we have introduced the interference effects with the following definitions:

$$c(\alpha,\widetilde{\alpha}) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} cos[\phi(t,\widetilde{\alpha}) - \phi(t,\alpha)]\,dt, \quad (8)$$

$$s(\alpha,\widetilde{\alpha}) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} sin[\phi(t,\widetilde{\alpha}) - \phi(t,\alpha)]\,dt, \quad (9)$$

$$c_I(\widehat{\alpha},\alpha,\Theta) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} cos[\phi(t,\alpha) - \phi(t,\widehat{\alpha}) - \Theta]\,dt, \quad (10)$$

$$c_I(\widehat{\alpha},\widetilde{\alpha},\Theta) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} cos[\phi(t,\widetilde{\alpha}) - \phi(t,\widehat{\alpha}) - \Theta]\,dt, \quad (11)$$

$$s_I(\widehat{\alpha},\alpha,\Theta) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} sin[\phi(t,\alpha) - \phi(t,\widehat{\alpha}) - \Theta]\,dt, \quad (12)$$

$$s_I(\widehat{\alpha},\widetilde{\alpha},\Theta) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} sin[\phi(t,\widetilde{\alpha}) - \phi(t,\widehat{\alpha}) - \Theta]\,dt, \quad (13)$$

$$n_c(\widetilde{\alpha}) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} n(t) cos[\omega_0 t + \phi(t,\widetilde{\alpha})]\,dt, \quad (14)$$

$$n_s(\widetilde{\alpha}) \triangleq \frac{1}{T}\int_{-N_1T}^{N_2T} n(t) sin[\omega_0 t + \phi(t,\widetilde{\alpha})]\,dt. \quad (15)$$

The parameter β is the quality of the phase estimate and is the ratio of the PLL SNR to the received SNR. The parameter β can take on non-negative real values that correspond to partially coherent detection ($0 \leq \beta \leq \infty$). Partially coherent detection reduces to coherent and noncoherent detection for values of $\beta = \infty$ and $\beta = 0$, respectively.

The absolute value of a complex Gaussian statistic is conditionally Rician, and the probability of one Rician exceeding another Rician, conditioned on the interference random variables, has been solved in [10] as

$$\begin{aligned} &Pr\left[R(\alpha,\tilde{\alpha_i}) > R(\alpha,\alpha) \mid \hat{\alpha},\Theta\right] \\ &\quad = \tfrac{1}{2}\left[1 - Q_M\left(\sqrt{b},\sqrt{a}\right) + Q_M\left(\sqrt{a},\sqrt{b}\right)\right], \end{aligned} \quad (16)$$

where $Q_M(\cdot,\cdot)$ is Marcum's Q-function

$$Q_M(x,y) = \int_y^\infty r \cdot e^{-(r^2+x^2)/2} \cdot I_0(x\cdot r)\, dr, \quad (17)$$

and the other terms are defined as

$$\left\{ \begin{array}{c} a \\ b \end{array} \right\} = \frac{1}{2\sigma^2}\left[\frac{|M_1|^2+|M_2|^2-2\sigma_1\sigma_2 Re\{M_2 M_1^* \rho\}}{1-|\rho|^2} \mp \frac{|M_2|^2-|M_1|^2}{\sqrt{1-|\rho|^2}}\right], \quad (18)$$

with the minus sign corresponding to the parameter "a" and the plus sign corresponding to the parameter "b". The terms comprising the parameters a and b are

$$\begin{aligned} &M_i = \mathbf{E}\{z_i\},\ i=1,2, \\ &\sigma^2 = var\{z_i\} = \tfrac{1}{2}\mathbf{E}\{(z_i - M_i)^*(z_i - M_i)\},\ i=1,2, \\ &\rho = \mathbf{E}\{(z_2 - M_2)^*(z_1 - M_1)\}/2\sigma^2, \end{aligned} \quad (19)$$

which are the mean, variance and the normalized cross correlation coefficient, respectively. $\mathbf{E}$ is the expectation operation and $*$ is the complex conjugate of a complex number. For $b >> 1$, $a >> 1$, and $\sqrt{b} >> \sqrt{b} - \sqrt{a} > 0$, it has been shown in [9] that the probability of one Rician exceeding another Rician can be approximated as

$$\begin{aligned} &Pr\left[R(\alpha,\tilde{\alpha}) > R(\alpha,\alpha) \mid \hat{\alpha},\Theta\right] \\ &\quad \approx Q\left(\sqrt{b}-\sqrt{a}\right) = Q\left(\sqrt{d_e^2(\hat{\alpha},\Theta)\tfrac{E}{N_0}}\right), \end{aligned} \quad (20)$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-v^2/2}dv$ is the normal Q-function, and $d_e(\hat{\alpha},\Theta) \triangleq (\sqrt{b}-\sqrt{a})/(E/N_0)$ is the normalized squared Euclidean distance. Care must be exercised when using the probability of error approximation Eqn. (20), because the parameters "a" and "b" are dependant upon the interference. It is found numerically that the approximation is good for an interference-to-signal power ratio (ISR) in the range [$-\infty$ dB, -10 dB] when the signal-to-noise power ratio (SNR) is in the range [5 dB, 15 dB].

Calculating the mean, variance, and normalized cross covariance, and using suitable simplifying terms, defined below, Eqn. (18) is rewritten for "a" and "b" as

$$\left\{ \begin{array}{c} a \\ b \end{array} \right\} = \frac{E}{2N_0}\left[\frac{\zeta_S+\zeta_I}{\zeta_D} \mp \frac{\eta_S+\eta_I}{\sqrt{\zeta_D}}\right]. \quad (21)$$

Substituting these terms into the normalized Euclidean distance expression results in

$$d_e^2(\hat{\alpha},\Theta) = \frac{(\eta_S+\eta_I)^2}{\zeta_S+\zeta_I+\sqrt{(\zeta_S+\zeta_I)^2-\zeta_D(\eta_S+\eta_I)^2}}, \quad (22)$$

where the subscript "S" corresponds to the effects of the desired signal, and the subscript "I" corresponds to the effects of the interfering signal. The definitions for the terms in Eqns. (21) and (22) are

$$\zeta_D \triangleq (N_1+N_2)^2 - \Delta^2(\alpha,\tilde{\alpha}), \quad (23)$$

$$\eta_S \triangleq (N_1+N_2)^2 - \Delta^2(\alpha,\tilde{\alpha}) + \beta\left[N_1+N_2-c(\alpha,\tilde{\alpha})\right], \quad (24)$$

$$\begin{aligned} \eta_I \triangleq\ & \left(\tfrac{I}{K}\right)\left\{\Delta_I^2(\hat{\alpha},\alpha,\Theta) - \Delta_I^2(\hat{\alpha},\tilde{\alpha},\Theta)\right\} \\ +\ & 2\sqrt{\tfrac{I}{K}}\left\{(N_1+N_2)\,c_I(\hat{\alpha},\alpha,\Theta)\right. \\ -\ & \left. c(\alpha,\tilde{\alpha})c_I(\hat{\alpha},\tilde{\alpha},\Theta) - s(\alpha,\tilde{\alpha})s_I(\hat{\alpha},\tilde{\alpha},\Theta)\right\} \\ +\ & \beta\sqrt{\tfrac{I}{K}}\left\{c_I(\hat{\alpha},\alpha,\Theta) - c_I(\hat{\alpha},\tilde{\alpha},\Theta)\right\}, \end{aligned} \quad (25)$$

$$\begin{aligned} \zeta_S \triangleq\ & (N_1+N_2+\beta)\left[(N_1+N_2)^2 - \Delta^2(\alpha,\tilde{\alpha})\right] \\ +\ & \tfrac{1}{2}\beta^2\left[N_1+N_2-c(\alpha,\tilde{\alpha})\right], \end{aligned} \quad (26)$$

and

$$\begin{aligned} \zeta_I \triangleq\ & \\ & \left(\tfrac{I}{K}\right)\Big[(N_1+N_2)\left\{\Delta_I^2(\hat{\alpha},\tilde{\alpha},\Theta) + \Delta_I^2(\hat{\alpha},\alpha,\Theta)\right\} \\ & \quad - 2c(\alpha,\tilde{\alpha})\left\{c_I(\hat{\alpha},\tilde{\alpha},\Theta)c_I(\hat{\alpha},\alpha,\Theta)\right. \\ & \qquad \left. + s_I(\hat{\alpha},\tilde{\alpha},\Theta)s_I(\hat{\alpha},\alpha,\Theta)\right\} \\ & \quad - 2s(\alpha,\tilde{\alpha})\left\{-c_I(\hat{\alpha},\tilde{\alpha},\Theta)s_I(\hat{\alpha},\alpha,\Theta)\right. \\ & \qquad \left. + s_I(\hat{\alpha},\tilde{\alpha},\Theta)s_I(\hat{\alpha},\alpha,\Theta)\right\}\Big] \\ & +2\sqrt{\tfrac{I}{K}}\Big[(N_1+N_2)\left\{c(\alpha,\tilde{\alpha})c_I(\hat{\alpha},\tilde{\alpha},\Theta)\right. \\ & \qquad \left. + s(\alpha,\tilde{\alpha})s_I(\hat{\alpha},\alpha,\Theta) + (N_1+N_2)c_I(\hat{\alpha},\alpha,\Theta)\right\} \\ & \quad - c(\alpha,\tilde{\alpha})\left\{c(\alpha,\tilde{\alpha})c_I(\hat{\alpha},\alpha,\Theta)\right. \\ & \qquad \left. + (N_1+N_2)c_I(\hat{\alpha},\tilde{\alpha},\Theta) + s(\alpha,\tilde{\alpha})s_I(\tilde{\alpha},\alpha,\Theta)\right\} \\ & \quad - s(\alpha,\tilde{\alpha})\left\{-c(\alpha,\tilde{\alpha})s_I(\hat{\alpha},\alpha,\Theta)\right. \\ & \qquad \left. + s(\alpha,\tilde{\alpha})c_I(\hat{\alpha},\alpha,\Theta) + (N_1+N_2)s_I(\hat{\alpha},\tilde{\alpha},\Theta)\right\}\Big] \\ & +\beta\sqrt{\tfrac{I}{K}}\Big[c_I(\hat{\alpha},\tilde{\alpha},\Theta) + c_I(\hat{\alpha},\alpha,\Theta) \\ & \quad - c(\alpha,\tilde{\alpha})\left\{c_I(\hat{\alpha},\tilde{\alpha},\Theta) + c_I(\hat{\alpha},\alpha,\Theta)\right\} \\ & \quad - s(\alpha,\tilde{\alpha})\left\{-s_I(\hat{\alpha},\alpha,\Theta) + s_I(\hat{\alpha},\tilde{\alpha},\Theta)\right\}\Big]. \end{aligned} \quad (27)$$

The expressions above have been simplified by using the following definitions:

$$\begin{aligned} \Delta^2(\alpha,\tilde{\alpha}) &\triangleq c^2(\alpha,\tilde{\alpha}) + s^2(\alpha,\tilde{\alpha}), \\ \Delta_I^2(\hat{\alpha},\tilde{\alpha},\Theta) &\triangleq c_I^2(\hat{\alpha},\tilde{\alpha},\Theta) + s_I^2(\hat{\alpha},\tilde{\alpha},\Theta), \\ \Delta_I^2(\hat{\alpha},\alpha,\Theta) &\triangleq c_I^2(\hat{\alpha},\alpha,\Theta) + s_I^2(\hat{\alpha},\alpha,\Theta). \end{aligned} \quad (28)$$

Again note that $\widetilde{\alpha}$ is the sequence estimate, $\widehat{\alpha}$ is the interference data sequence, and α is the true transmitted data sequence.

When the ratio of signal-to-noise power is reasonably high, it turns out that one error sequence completely dominates the probability of symbol error and is referred to as a Type 1 error sequence in [9]. Given the sequences

$$\alpha = \{\alpha_{-N_1}, \alpha_{-N_1+1}, \cdots, \alpha_{N_2-1}, \alpha_{N_2}\} \qquad (29)$$

and

$$\widetilde{\alpha} = \{\widetilde{\alpha}_{-N_1}, \widetilde{\alpha}_{-N_1+1}, \cdots, \widetilde{\alpha}_{N_2-1}, \widetilde{\alpha}_{N_2}\}, \qquad (30)$$

the Type 1 error sequence has α and $\widetilde{\alpha}$ only differing in the i=0 and i=1 positions and has the property that

$$\widetilde{\alpha}_0 - \alpha_0 = -(\widetilde{\alpha}_1 - \alpha_1). \qquad (31)$$

Other error sequences result in larger normalized square Euclidean distances and hence smaller probabilities, so they are ignored here.

The probability of error is then averaged over both all possible interference sequences and the random interference phase offset. This can be written as

$$\begin{aligned} Pr[error] &= P_e \\ &= \sum_{i=1}^{2^{N_1+N_2}} Pr(\widehat{\alpha} = \alpha_i) \int_{-\pi}^{\pi} Q\left(\sqrt{d_e^2(\alpha_i, \Theta)\frac{E}{N_0}}\right) \frac{d\Theta}{2\pi}. \end{aligned} \qquad (32)$$

Assuming equally likely interference sequences further simplifies this expression to

$$P_e = \frac{1}{2^{N_1+N_2}} \sum_{i=1}^{2^{N_1+N_2}} \int_{-\pi}^{\pi} Q\left(\sqrt{d_e^2(\alpha_i, \Theta)\frac{E}{N_0}}\right) \frac{d\Theta}{2\pi}. \qquad (33)$$

IV. NUMERICAL RESULTS AND DISCUSSION

Fig. 2 is a plot of the probability of error, P_e, vs. the modulation index, h, for noncoherent detection ($\beta = 0$) of binary data using Eqn. (33) with a 1REC phase function observed over the three symbol interval [-T,2T]. The probability of error for partially coherent detection is shown in Fig. 3, with a suitably chosen coherent quality parameter of $\beta = 5$, and can be seen to be an improvement over the noncoherent detector performance of Fig. 2. Coherent reception is shown in Fig. 4 and has the best performance for the same three symbol observation interval. All of the curves in these figures are indexed by the various interference-to-signal power ratios (ISR). The minimum values of the curves in these figures correspond to the best probability of error performance.

As expected, the partially coherent performance, although never as good as the coherent receiver, is superior to the noncoherent receiver over a large range of β. Notice in these figures the distinctive minimum troughs for modulation index values in the range of 0.5 to 1.0. These minima correspond to the minimum probability of error for the given ISR and SNR values. The minimum value of these troughs is the desired operating point, but these minima also correspond to

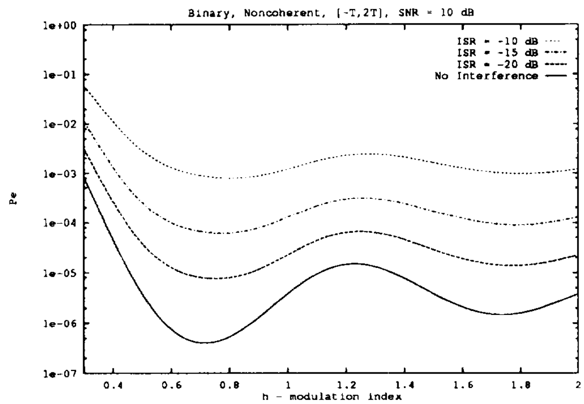

Figure 2: Probability of bit error for noncoherent detection with different interference powers using 1REC phase functions.

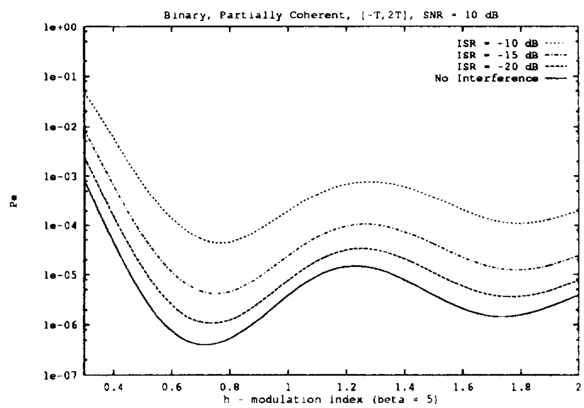

Figure 3: Probability of bit error for partially coherent detection ($\beta = 5$) with different interference powers using 1REC phase functions.

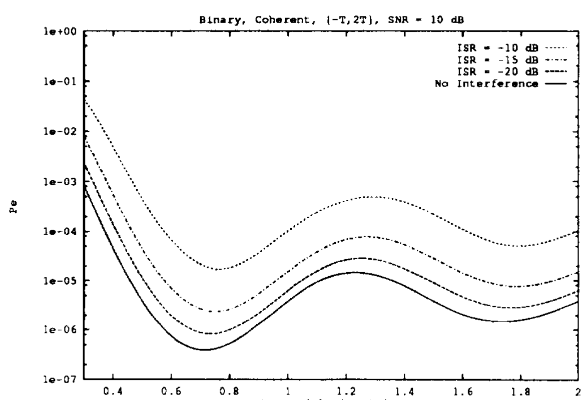

Figure 4: Probability of bit error for coherent detection with different interference powers using 1REC phase functions.

the highest system degradation. The system degradation is defined as the amount the SNR may be reduced when the interference is removed for the same system probability of error performance.

A summary of the modulation indices, h, that give the minimum probability of errors, P_e, and the corresponding degradation for detection over three symbols, is shown in Table 1. The modulation indices for minimum Pe increase as the interference power increases. In Table 1, all of the modulation indices for the best bit error rate are less than or equal to one.

	Minimum Pe		
ISR (dB)	h	Pe (min.)	Degrade (dB)
Noncoherent:			
-∞	0.715	4.018×10^{-7}	0
-20	0.754	7.642×10^{-6}	1.353
-15	0.774	6.171×10^{-5}	2.874
-10	0.801	8.019×10^{-4}	6.097
Part. Coh. ($\beta = 5$)			
-∞	0.715	4.018×10^{-7}	0
-20	0.737	1.089×10^{-6}	0.376
-15	0.753	4.173×10^{-6}	1.006
-10	0.767	4.357×10^{-5}	2.552
Coherent:			
-∞	0.715	4.018×10^{-7}	0
-20	0.734	8.382×10^{-7}	0.264
-15	0.751	2.398×10^{-6}	0.711
-10	0.765	1.748×10^{-5}	1.766

Table 1: Optimum modulation index for minimum probability of error performance and corresponding degradation with SNR = 10 dB, 1REC, and observation over three symbol intervals [-T,2T].

The variation of the minimum modulation index over ISR values is greatest for noncoherent detection and least for coherent detection. The variation of the minimum h for partially coherent reception is close to that of the coherent detector even with a β as small as five. The P_e performance of a partially coherent receiver is likewise close to that of the coherent receiver. Consequently, even better performance is expected for larger values of the coherence quality parameter β. The partially coherent receiver is then seen as a way to allow data detection regardless of the state of the PLL, provided some measure of the quality of the PLL phase estimate is used.

V. CONCLUSION

The analysis of a partially coherent receiver with a single cochannel interfering signal was performed. Using a small value of the phase coherence quality parameter, β, significant improvements over the noncoherent receiver are obtained. When an interfering signal is present, the optimum modulation index for the best probability of error performance increases as a function of the interference strength. The variation with ISR of the optimum modulation index, h, and probability of error, P_e, is less for the partially coherent receiver than for the noncoherent receiver, and is close to that of the coherent receiver.

VI. ACKNOWLEDGEMENTS

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